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Transient Coupled Radiation–Conduction in Semitransparent Spherical Particle

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A method to analyze transient coupled radiation–conduction in a semitransparent spherical particle surrounded by isothermal black walls was developed. The radiative transfer coefficients were deduced using the ray tracing method in combination with Hottel and Sarofim's zonal method. The radiative heat source term was calculated by the radiative transfer coefficients, and the transient energy equation was solved by an implicit finite difference method. The effects of the related parameters on the transient radiative heat source and temperature distribution were analyzed. The results show that the peak of dimensionless radial radiative heat source can locate in the interior shell of the particle or droplet with small optical thickness when heated by surrounding radiation. Treating volume radiation as surface radiation results in large transient temperature distribution errors for the particle with small optical thickness.

Nomenclature

C_p = specific heat
E = directional spectral radiative energy
H_i = radiative heat source of volume i, $4\pi r_i^2 \Delta r h_i$
h = radiative heat source
I = radiative intensity
k = thermal conductivity
m = complex index of refraction, $n - i\kappa$
N = conduction-to-radiation parameter, $k\beta/(4n^2\sigma T_0^3)$
n = refractive index
Q = radiative heat absorption
R = outside radius of particle
r = radial coordinate
$[SV_i]_\lambda$ = spectral radiation transfer coefficients of surrounding vs volume i
T = temperature
T_{sur} = temperature of surroundings
T_0 = initial temperature
t = time
u, v = coefficient defined in Eq. (5)
$[V_i V_j]_\lambda$ = spectral radiation transfer coefficients of volume i vs volume j
x = size parameter, $2\pi R/\lambda$
x_{max} = size parameter based on the wavelength λ_{max}, $2\pi R/\lambda_{max}$
y = dimensionless radial coordinate, r/R
α_λ = spectral absorptivity defined in Eq. (32)
β = absorption coefficient, $4\pi\kappa/\lambda_{max}$
Θ = dimensionless temperature, T/T_0
θ = angle
κ = index of absorption
λ = wavelength in vacuum
λ_{max} = wavelength corresponding to the maximum spectral emissive power at the surrounding temperature T_{sur}
ξ = dimensionless time, $k\beta^2 t/(\rho C_p)$
ρ = density of particle substance,
$\rho_\parallel$ = parallel polarized component of reflectivity
$\rho_\perp$ = perpendicular polarized component of reflectivity
τ = optical thickness variable, βr
τ_0 = optical thickness of particle, βR
Φ = dimensionless radiative heat source, $h/(4n^2\beta\sigma T_0^4)$
ϕ = angle
ω = angle
ω_1 = angle defined in Eq. (21)

Subscripts

b = blackbody
i = volume and node i
λ = at a given wavelength
$\parallel$ = parallel polarized component
$\perp$ = perpendicular polarized component

Superscript

m = time step

Introduction

TRANSIENT coupled radiation and conduction in a semitransparent spherical medium is one of the pervasive processes in engineering applications, such as spray combustion,[1–3] spray cooling,[4] spray drying, spray forming,[5] and so on. Temperature distributions within semitransparent materials can be strongly affected by internal emission and absorption of radiant thermal energy. This is important for translucent materials at elevated temperatures, in high-temperature surroundings, or with large incident radiation. Because of its complexity, thermal radiation absorption of particles was often considered to behave like bulk materials; consequently, the absorption process was treated strictly as a surface interaction. In the spray processing of a semitransparent medium, the transient behavior of particles or droplets must be examined because translucence can result in internal temperature responses that are much more rapid and that have different distributions than by heat conduction alone.

Recently, much attention has been focused on the determining of the transient effects of radiation and conduction heat transfer in semitransparent materials. Siegel's detailed review[6] of the literature

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surrounding the subject over the past 40 years found that despite the relatively large amount of interest in transient coupled radiation and conduction in a semitransparent medium, most of the work has focused on slab geometrical systems. Only a limited amount of research is available on transient coupled radiation and conduction in semitransparent spherical geometrical systems. Sitarski[3] studied heat transport inside an irradiated slurry droplet. In this research, the transient energy equation was solved by a finite difference method, and the radiant heat source was calculated using the Mie scattering theory. Tsai and Ozisik[7] considered combined transient conduction and radiation in an absorbing, emitting, and isotropically scattering solid sphere with a black boundary. An implicit finite difference method was applied to obtain temperature distributions from the transient energy equation including heat conduction. The collocation method was used for evaluating the radiative source distribution.

A transient solution of coupled radiation–conduction involves two parts: the evaluation of the spatial distribution of the radiative energy source at each time and the solution of the transient energy equation. For transient heat transfer problems in the semitransparent particles or droplets, the key difficulty is in determining the local radiant source term in the transient energy equation. Lage and Rangel[1] and Tuntomo et al.[8] applied electromagnetic theory to study the internal radiant absorption field of a small spherical particle. Dombrovsky and Ignat'ev[9] studied the temperature distribution of large semitransparent particles of a material of low thermal conductivity during their motion in a gas jet. The radiation inside a particle was calculated by use of the geometrical optics approximation in combination with the modified differential approximation. Based on electromagnetic theory, Mackowski et al.[10] presented series expressions for the radial dependent absorption cross section and heat source function in a stratified sphere. Dombrovsky[11] determined the radial profile of the radiation heat source inside a particle not only by use of the Mie theory but also by use the radiation transfer equation. A comparison with the Mie theory calculations for isothermal particles shows that the geometrical optics approximation is sufficiently accurate, even for particles that are not very large (size parameter is greater than 20), both for the particle emissivity value and for the heat generation profile. Generally, the local radiant heat source within single spheres can be handled by electromagnetic theory or Mie theory (see Ref. 12). However, computation using electromagnetic theory or Mie theory is complex and time consuming. Mie's theory can be approximated with ray optics when the sphere diameter is much larger than the wavelength.

Tan and Lallemand[13] studied coupled radiation and conduction in a glass slab using the ray tracing method in combination with Hottel and Sarofim's zonal method.[14] Recently, Wang et al.[15] extended this method to study the coupled radiation and conduction in a semitransparent composite slab. Their studies showed that this method is accurate, and the key to the method is in determining the radiative transfer coefficients.

Because of the different geometrical features of a sphere and a slab, the behavior of transient coupled conduction–radiation within a semitransparent spherical particle differs from a slab's. The objective of the present work is to extend the ray tracing method and the zonal method to study transient coupled conduction–radiation within semitransparent particles. The particle is surrounded by isothermal black wall, and the space between the particle and the surrounding wall is a vacuum. The radiative transfer coefficients are deduced using the ray tracing method in combination with Hottel and Sarofim's zonal method.[14] The radiative heat source term is calculated by the radiative transfer coefficients, and the transient energy equation is solved by an implicit finite difference method. The effects of the related parameters on the transient radiant heat source and temperature distribution are analyzed.

Physical Model and Formulation

Physical Model

We consider a semitransparent spherical particle with radius R. The particle is surrounded by isothermal black wall, and the space between the particle and the surrounding wall is a vacuum. The particle is initially at a uniform temperature T_0. In addition, the following assumptions are made in analysis:

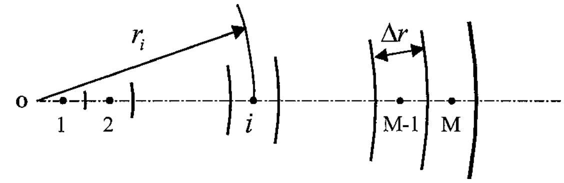

Fig. 1 Grid system.

1) The particle is composed of an isotropic and homogeneous medium.
2) The medium emits and absorbs but does not scatter thermal radiation.
3) The particle surface is optically smooth.
4) The complex index of refraction $m = n - i\kappa$, density ρ, specific heat C_p, and conductivity k of medium does not depend on the temperature.
5) The complex reflective index does not depend on wavelength.

The energy equation for transient coupled radiative and conductive heat transfer in a semitransparent particle is given by

$$\rho C_p \frac{\partial T(r,t)}{\partial t} = \frac{k}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial T(r,t)}{\partial r}\right] + h(r,t) \tag{1a}$$

with the boundary and initial conditions, at $r = 0$,

$$\frac{\partial T(r,t)}{\partial r} = 0 \tag{1b}$$

at $r = R$,

$$\frac{\partial T(r,t)}{\partial r} = 0 \tag{1c}$$

and at $t = 0$,

$$T(r,t) = T_0 \tag{1d}$$

The semitransparent particle is divided into M uniform control volume elements in a radial direction as shown in Fig. 1. Using an implicit central difference approximation, a control volume form of the energy equation can be obtained by considering the integration of Eq. (1a) over a mesh cell within one-dimensional spherical geometry. The fully implicit discretized energy equation of the control volume i is obtained as

$$\frac{4\pi\rho C_p r_i^2 \Delta r}{\Delta t}\left(T_i^{m+1} - T_i^m\right) = \frac{4\pi k r_{i+\frac{1}{2}}^2 \left(T_{i+1}^{m+1} - T_i^{m+1}\right)}{\Delta r} + \frac{4\pi k r_{i-\frac{1}{2}}^2 \left(T_{i-1}^{m+1} - T_i^{m+1}\right)}{\Delta r} + H_i^{m+1} \qquad 2 \le i \le M-1 \tag{2a}$$

$$\frac{4\pi\rho C_p r_i^2 \Delta r}{\Delta t}\left(T_i^{m+1} - T_i^m\right) = \frac{4\pi k r_{i+\frac{1}{2}}^2 \left(T_{i+1}^{m+1} - T_i^{m+1}\right)}{\Delta r} + H_i^{m+1} \qquad i = 1 \tag{2b}$$

$$\frac{4\pi\rho C_p r_i^2 \Delta r}{\Delta t}\left(T_i^{m+1} - T_i^m\right) = \frac{4\pi k r_{i-\frac{1}{2}}^2 \left(T_{i-1}^{m+1} - T_i^{m+1}\right)}{\Delta r} + H_i^{m+1} \qquad i = M \tag{2c}$$

where H_i is the local radiative heat source term given by

$$H_i = 4\pi r_i^2 \Delta r h_i \tag{2d}$$

Radiative Heat Source Term

The key to solving the transient energy equation is to compute the local radiative heat source term H_i. We use the ray tracing method to deduce the detailed computation formulas for the radiative heat source term. The local radiative heat source term within a semitransparent particle includes three parts: 1) absorption of surroundings

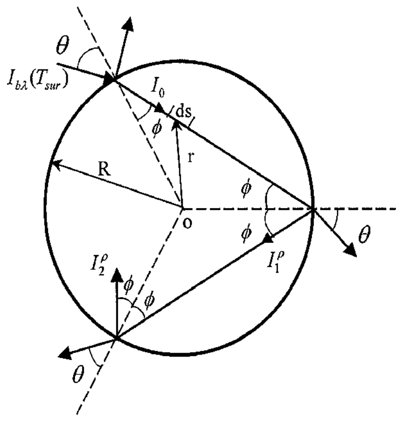

Fig. 2 Ray tracing of radiation emitted by the surroundings.

irradiation, 2) absorption of internal radiation, and 3) internal self-emission.

Consider irradiation of a spherical particle with incidence angle θ. As shown in Fig. 2, the refraction angle ϕ is related to the incidence angle and the complex refractive index through Snell's law as

$$\sin\phi = (1/n)\sin\theta \tag{3}$$

The reflectivities of external specular reflection are[16,17]

$$\rho_\perp(\theta) = \frac{(\cos\theta - u)^2 + v^2}{(\cos\theta + u)^2 + v^2} \tag{4a}$$

$$\rho_\parallel(\theta) = \frac{(u\cos\theta - \sin^2\theta)^2 + v^2\cos^2\theta}{(u\cos\theta + \sin^2\theta)^2 + v^2\cos^2\theta}\rho_\perp(\theta) \tag{4b}$$

where $\rho_\perp$ and $\rho_\parallel$ are perpendicular and parallel polarized components, respectively, and u and v are the coefficients given by[16,17]

$$u^2 = 0.5\{[(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2]^{0.5} + (n^2 - \kappa^2 - \sin^2\theta)\} \tag{5a}$$

$$v^2 = 0.5\{[(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2]^{0.5} - (n^2 - \kappa^2 - \sin^2\theta)\} \tag{5b}$$

To derive the computational formulas of the local absorption of surroundings irradiation, we trace an incident ray from the surroundings with an incidence angle θ. The ray enters the spherical particle with the initial refraction angle ϕ. After entering the particle, the ray undergoes multiple reflections and refractions. Through symmetry, ϕ becomes the incident angle for all internal reflections, and $2R\cos\phi$ is the distance traveled between consecutive surface encounters. The reflectivities of internal specular reflection are equal to that of external specular reflection if expressed in term of the angle θ. For the unpolarized incident radiation from the surroundings, the relation between the radiative intensities of the initial incident and refracted rays is determined by energy equilibrium as

$$0.5[1 - \rho_\perp(\theta)]I_{b\lambda}(T_{sur})\cos\theta 2\pi\sin\theta\,d\theta = I_{\perp,0}^{sur}\cos\phi 2\pi\sin\phi\,d\phi \tag{6a}$$

$$0.5[1 - \rho_\parallel(\theta)]I_{b\lambda}(T_{sur})\cos\theta 2\pi\sin\theta\,d\theta = I_{\parallel,0}^{sur}\cos\phi 2\pi\sin\phi\,d\phi \tag{6b}$$

where $I_{b\lambda}(T_{sur})$ is the spectral radiative intensity of the incident ray from the surroundings and $I_{\perp,0}^{sur}$ and $I_{\parallel,0}^{sur}$ are the perpendicular and parallel polarized components of radiative intensity of the initial refracted ray, respectively. They are rewritten as

$$I_{\perp,0}^{sur} = 0.5[1 - \rho_\perp(\theta)]I_{b\lambda}(T_{sur})\frac{\sin\theta\cos\theta\,d\theta}{\sin\phi\cos\phi\,d\phi} \tag{7a}$$

$$I_{\parallel,0}^{sur} = 0.5[1 - \rho_\parallel(\theta)]I_{b\lambda}(T_{sur})\frac{\sin\theta\cos\theta\,d\theta}{\sin\phi\cos\phi\,d\phi} \tag{7b}$$

For the perpendicular component, after the first internal reflection, spectral radiative intensity of the reflected ray becomes

$$\begin{aligned} I_{\perp,1}^{sur,\rho} &= I_{\perp,0}^{sur}\rho_\perp(\theta)\exp[-(4\pi\kappa/\lambda)2R\cos\phi] \\ &= I_{\perp,0}^{sur}\rho_\perp(\theta)\exp(-4x\kappa\cos\phi) \end{aligned} \tag{8}$$

where x is the size parameter defined as

$$x = 2\pi R/\lambda \tag{9}$$

After the kth internal reflection, the perpendicular spectral radiative intensity component of the reflected ray is given as

$$I_{\perp,k}^{sur,\rho} = I_{\perp,0}^{sur}[\rho_\perp(\theta)\exp(-4x\kappa\cos\phi)]^k \tag{10}$$

During the kth and $(k+1)$th internal reflection, the perpendicular spectral radiative intensity component absorbed by the sphere with radius r in the particle is written as

$$\begin{aligned} \Delta I_{\perp,k}^{sur,a} &= I_{\perp,0}^{sur}[\rho_\perp(\theta)\exp(-4x\kappa\cos\phi)]^k \\ &\times \left\{\exp\left[-2x\kappa\left(\cos\phi - \sqrt{y^2 - \sin^2\phi}\right)\right]\right. \\ &\left. - \exp\left[-2x\kappa\left(\cos\phi + \sqrt{y^2 - \sin^2\phi}\right)\right]\right\} \end{aligned} \tag{11}$$

where y is the dimensionless radius defined as

$$y = r/R \tag{12}$$

The absorption of the perpendicular spectral radiative intensity component in the infinite series of internal reflection traversals of a single ray is summed as

$$\Delta I_\perp^{sur,a} = \sum_{k=0}^{\infty}\Delta I_{\perp,k}^{sur,a} = \frac{I_{\perp,0}^{sur}\left\{\exp\left[-2x\kappa\left(\cos\phi - \sqrt{y^2 - \sin^2\phi}\right)\right] - \exp\left[-2x\kappa\left(\cos\phi + \sqrt{y^2 - \sin^2\phi}\right)\right]\right\}}{1 - \rho_\perp(\theta)\exp(-4x\kappa\cos\phi)} \tag{13}$$

Similarly, the absorption of the parallel spectral radiative intensity component in the infinite series of internal reflection traversals of a single ray is written as

$$\Delta I_\parallel^{sur,a} = \sum_{k=0}^{\infty}\Delta I_{\parallel,k}^{sur,a} = \frac{I_{\parallel,0}^{sur}\left\{\exp\left[-2x\kappa\left(\cos\phi - \sqrt{y^2 - \sin^2\phi}\right)\right] - \exp\left[-2x\kappa\left(\cos\phi + \sqrt{y^2 - \sin^2\phi}\right)\right]\right\}}{1 - \rho_\parallel(\theta)\exp(-4x\kappa\cos\phi)} \tag{14}$$

After integrating over the total surface of the particle and the range of the solid angle while considering the symmetry, we derive the following relation for spectral radiation absorption of a sphere with dimensionless radius y in the particle:

$$Q^{\rm sur}(y,\lambda)=\int_0^{\phi_{\rm end}}4\pi R^2\left(\Delta I_\perp^{{\rm sur},a}+\Delta I_\parallel^{{\rm sur},a}\right)\cos\phi\, 2\pi\sin\phi\,{\rm d}\phi \tag{15}$$

Substituting Eqs. (7a) and (7b) and Eqs. (13) and (14) for Eq. (15), we have

$$\begin{aligned}Q^{\rm sur}(y,\lambda)&=4\pi^2R^2I_{b\lambda}(T_{\rm sur})\\&\times\int_0^{\theta_{\rm end}}\left\{\exp\left[-2x\kappa\left(\cos\phi-\sqrt{y^2-\sin^2\phi}\right)\right]\right.\\&\left.-\exp\left[-2x\kappa\left(\cos\phi+\sqrt{y^2-\sin^2\phi}\right)\right]\right\}\\&\times\left\{\frac{1-\rho_\perp(\theta)}{1-\rho_\perp(\theta)\exp(-4x\kappa\cos\phi)}+\frac{1-\rho_\parallel(\theta)}{1-\rho_\parallel(\theta)\exp(-4x\kappa\cos\phi)}\right\}\\&\times\sin\theta\cos\theta\,{\rm d}\theta\end{aligned}\tag{16}$$

Here, $\theta_{\rm end}$ is determined as follows:

$$\theta_{\rm end}=0.5\pi,\qquad \text{if}\quad 1/n<y\le 1 \tag{17a}$$

$$\theta_{\rm end}=\sin^{-1}(ny),\qquad \text{if}\quad 0\le y\le 1/n \tag{17b}$$

From Eq. (16), the spectral radiation transfer coefficient of surrounding vs volume i is expressed as

$$[SV_i]_\lambda=\frac{Q^{\rm sur}(y_i+0.5\Delta y_i,\lambda)-Q^{\rm sur}(y_i-0.5\Delta y_i,\lambda)}{\pi I_{b\lambda}(T_{\rm sur})} \tag{18}$$

To derive the computational formulas of the local absorption of internal radiation, we trace an emitting ray from the differential volume ${\rm d}A_i\,\Delta r_i$ at site A of the sphere surface with radius r_i. Here, ${\rm d}A_i$ is the differential area in the surface of the sphere with radius r_i. As shown in Fig. 3, the ray hits the particle surface with the incidence angle ϕ. After that, the ray undergoes multiple reflection and refraction. The incidence angle ϕ is related to the emittance angle ω as

$$\phi=\sin^{-1}(y_i\sin\omega) \tag{19}$$

If $r_j\ge r_i$, any rays emitted by site A of sphere surface with radius r_i will intersect with the sphere with radius r_j. The length of the path $\overline{AC}$ is given by

$$S_{AC}=R(y_j\cos\omega_1-y_i\cos\omega) \tag{20}$$

where the angle ω_1 is given by

$$\omega_1=\sin^{-1}(y_i\sin\omega/y_j) \tag{21}$$

The directional spectral radiative energy absorbed by the spherical shell with thickness Δr_j at the location of $r=r_j$ is expressed as

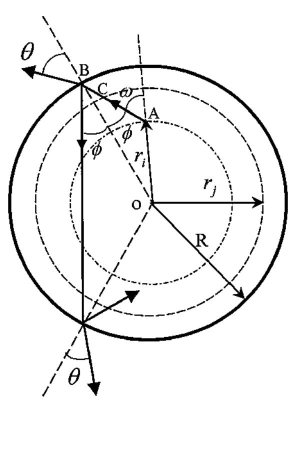

Fig. 3 Ray tracing of radiation emitted by the media within the particle.

$$\begin{aligned}\Delta E_0^{\rm in}&=\left(\frac{4\pi\kappa}{\lambda}\right)^2n^2I_{b\lambda}(T_i)\,{\rm d}A_i\,\Delta r_i\exp\left(-\frac{4\pi\kappa}{\lambda}S_{AC}\right)\frac{{\rm d}S_{AC}}{{\rm d}r_j}\Delta r_j\\&=4(xn\kappa)^2I_{b\lambda}(T_i)\exp[-2x\kappa(y_j\cos\omega_1-y_i\cos\omega)]\\&\times\frac{y_j\,\Delta y_j}{\sqrt{y_j^2-\sin^2\phi}}\,{\rm d}A_i\,\Delta y_i\end{aligned}\tag{22}$$

Before undergoing multiple reflection and refraction, the ray emitted by the differential volume ${\rm d}A_i\,\Delta r_i$ reaches the particle's internal surface with the directional spectral radiative energy E_0 given by

$$E_0=\frac{4x\kappa}{\lambda}n^2I_{b\lambda}(y_i)\,{\rm d}A_i\,\Delta r_i\exp\left[-\frac{4\pi\kappa}{\lambda}S_{AC}\right] \tag{23}$$

When Eqs. (19–21) and (23) are combined, Eq. (23) can be rewritten as

$$E_0=\frac{4x\kappa}{\lambda}n^2I_{b\lambda}(y_i)\,{\rm d}A_i\,\Delta r_i\exp[-2x\kappa(\cos\phi-y_i\cos\omega)] \tag{24}$$

Similar to the absorption of the radiation of the surroundings, during multiple reflections and refractions, the directional spectral radiative energy absorbed by the spherical shell with thickness Δr_j at the location of $r=r_j$ is expressed as

$$\begin{aligned}\Delta E_{\rm multi}^{\rm in}&=4(n x\kappa)^2I_{b\lambda}(T_i)\left\{\exp\left[-2x\kappa\left(\cos\phi-\sqrt{y_j^2-\sin^2\phi}\right)\right]\right.\\&\left.+\exp\left[-2x\kappa\left(\cos\phi+\sqrt{y_j^2-\sin^2\phi}\right)\right]\right\}\\&\times 0.5\left\{\frac{\rho_\perp(\theta)}{1-\rho_\perp(\theta)\exp(-4x\kappa\cos\phi)}\right.\\&\left.+\frac{\rho_\parallel(\theta)}{1-\rho_\parallel(\theta)\exp(-4x\kappa\cos\phi)}\right\}\\&\times\exp[-2x\kappa(\cos\phi-y_i\cos\omega)]\frac{y_j\,\Delta y_j}{\sqrt{y_j^2-\sin^2\phi}}\,{\rm d}A_i\,\Delta y_i\end{aligned}\tag{25}$$

The relation between the angles ϕ and θ is determined by Snell's law. For the reflection of the ray emitted by internal medium of particle, the angle for total reflection, ϕ_{cr}, is defined as

$$\phi_{cr}=\sin^{-1}(1/n) \tag{26}$$

If $\phi<\phi_{cr}$, the reflectivities used in Eq. (25) can be calculated by Eqs. (4a) and (4b); otherwise, $\rho_\perp=\rho_\parallel=1$. Summing Eqs. (22) and (25), integrating the sum over the total surface of the sphere with the radius r_i and the range of the solid angle, and dividing it by $\pi I_{b\lambda}(T_i)$, we derive the following relation for the spectral radiation transfer coefficient of volume i vs volume j as

$$\begin{aligned}[V_iV_j]_\lambda&=32\pi R^2\int_{\omega=0}^{\omega=\pi}\left(\left\{\exp\left[-2x\kappa\left(\cos\phi-\sqrt{y_j^2-\sin^2\phi}\right)\right]\right.\right.\\&\left.+\exp\left[-2x\kappa\left(\cos\phi+\sqrt{y_j^2-\sin^2\phi}\right)\right]\right\}\\&\times 0.5\left[\frac{\rho_\perp(\theta)}{1-\rho_\perp(\theta)\exp(-4x\kappa\cos\phi)}\right.\\&\left.+\frac{\rho_\parallel(\theta)}{1-\rho_\parallel(\theta)\exp(-4x\kappa\cos\phi)}\right]\\&\times\exp[-2x\kappa(\cos\phi-y_i\cos\omega)]\\&\left.+\exp[-2x\kappa(y_j\cos\beta_1-y_i\cos\omega)]\right)\\&\times\frac{y_j\,\Delta y_j}{\sqrt{y_j^2-\sin^2\phi}}y_i^2\,\Delta y_i\,(nx\kappa)^2\sin\omega\,{\rm d}\omega\end{aligned}\tag{27}$$

According to the relativities of the spectral radiative transfer coefficients, we have

$$[V_i V_j]_\lambda = [V_j V_i]_\lambda \tag{28}$$

Using the spectral radiative transfer coefficients, the radiative heat source term of the control volume i is expressed as

$$H_i = \int_0^\infty \left\{ \pi I_{b\lambda}(T_{\text{sur}})[SV_i]_\lambda + \sum_{j=1}^{M} \pi I_{b\lambda}(T_j)[V_j V_i]_\lambda - 16\pi^2 r_i^2 \Delta r_i \left(\frac{4\pi\kappa}{\lambda}\right) n^2 I_{b\lambda}(T_i) \right\} \mathrm{d}\lambda \tag{29}$$

where the third term on the right side of Eq. (29) is the self-emission of the volume i.

Method for Solution

Equations (1–29) provide the complete mathematical formulation for the problem of transient coupled radiation and conduction in a one-dimensional semitransparent particle. An iterative process is needed to solve the problem because the energy equation involves the radiative heat source term, whereas the radiative heat source term requires the temperature distribution. Equation (29) for the radiative heat source term was approached by numerical integration, and the discretized energy equation (2a) was solved by the tridiagonal matrix algorithm method. At each time step, the temperature convergence criteria was taken as follows:

$$\max\left|\frac{T^{\text{new}} - T^{\text{old}}}{T^{\text{new}}}\right| < 10^{-6} \tag{30}$$

where superscripts new and old denote the present and the previous iteration values, respectively.

Because of its complexity, thermal radiation absorption of particles was often considered to behave like bulk materials, and consequently, the absorption process was treated strictly as a surface interaction. For comparison, the transient heat transfer process, in which the volume radiation is considered as a surface interaction, was solved numerically. The corresponding transient energy equation is written as

$$\rho C_p \frac{\partial T(r,t)}{\partial t} = \frac{k}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial T(r,t)}{\partial r}\right] \tag{31a}$$

with the boundary and initial conditions, at $r = 0$,

$$\frac{\partial T(r,t)}{\partial r} = 0 \tag{31b}$$

at $r = R$,

$$-k\frac{\partial T(r,t)}{\partial r} = \int_0^\infty \alpha_\lambda \pi [I_{b\lambda}(T_w) - I_{b\lambda}(T_{\text{sur}})]\,\mathrm{d}\lambda \tag{31c}$$

and at $t = 0$,

$$T(r,t) = T_0 \tag{31d}$$

where the spectral absorptivity α_λ is defined as

$$\alpha_\lambda = \frac{Q^{\text{sur}}(1,\lambda)}{4\pi^2 R^2 I_{b\lambda}(T_{\text{sur}})} \tag{32}$$

From Eqs. (16) and (32), it can be seen that, if the complex refractive index is independent on the temperature, the spectral absorptivity α_λ is a function of n, κ, and x and is independent of the temperature of the particle and the surroundings. An implicit central difference scheme was used to solve Eq. (31a).

Results and Discussion

A computer code based on the preceding calculation procedure was written. Grid refinement and time step sensitivity studies were also performed for the physical model to ensure that the essential physics are independent of grid size and time interval. To validate the formulas derived in this paper to some extent, we calculated the total spectral absorptance α_λ by Mie theory and ray tracing method, respectively. The results of Mie theory are computed by using the program BHMIE given in the Appendix A of Ref. 12. Comparisons of these results are shown in Fig. 4. When the size parameter of the particle is greater than 30, the results of the ray tracing method is close to that of Mie theory, and the Mie theory can be approximated accurately with the geometrical optics if the size parameter of the particle is greater than 400. This proves the formulas for radiative source term derived in this paper to be correct. Because the geometrical optics can approximate Mie theory well for larger particles, in this paper, we consider only the transient coupled radiation–conduction processes in the particle with size parameter $x_{\max} \geq 50$. The size parameter $x_{\max}$ is based on the wavelength $\lambda_{\max}$ at which the spectral blackbody emissive power is a maximum for a given temperature T_{sur} of the surroundings.

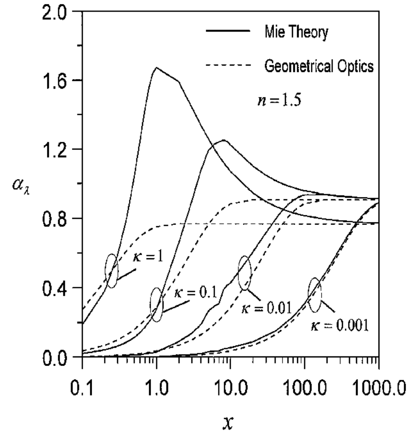

Fig. 4 Comparison of spectral absorptivities calculated by the Mie theory and the geometrical optics.

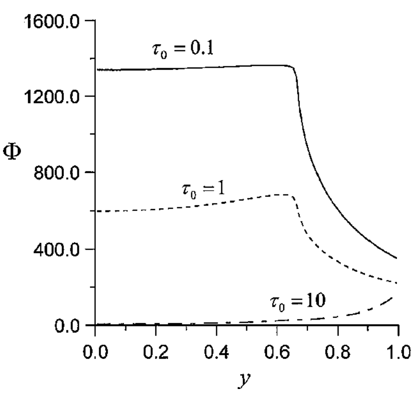

Fig. 5 Influences of optical thickness on the dimensionless radiative heat source distribution in the case of $N = 1$, $n = 1.5$, $\kappa = 0.001$, $T_{\text{sur}} = 2000$ K, $T_0 = 300$ K, and $\xi = 0.002$.

From Eqs. (16), (27), and (29), we can see that the size parameter and the complex refractive index of the particle affects the radial distribution profiles of the radiative heat source inside the particle; hence, it affects the transient heat transfer processes. The formulas deduced in this paper are general and can be used to compute the radiative heat source for the case of variable complex refractive index, but only the results bound by the assumption of constant complex refractive index are discussed.

Figure 5 shows the dimensionless radial radiative heat source distributions within a semitransparent particle for three different particle optical thicknesses in the case of $N = 1$, $n = 1.5$, $\kappa = 0.001$, $T_{\text{sur}} = 2000$ K, $T_0 = 300$ K, and $\xi = 0.002$. From Fig. 5, we can find the peak dimensionless radial radiative heat source is located in the interior shell of the particle or droplet with small optical thickness. This phenomenon may be related to the internal burst or overheating

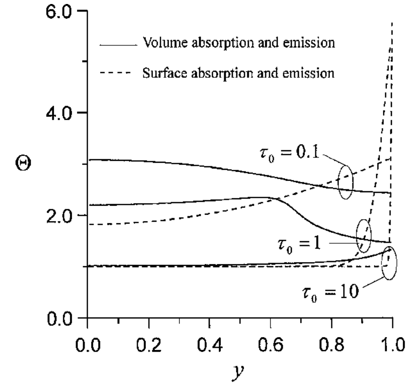

Fig. 6 Influence of optical thickness on the dimensionless temperature distribution in the case of $N = 1$, $n = 1.5$, $\kappa = 0.001$, $T_{sur} = 2000$ K, $T_0 = 300$ K, and $\xi = 0.002$.

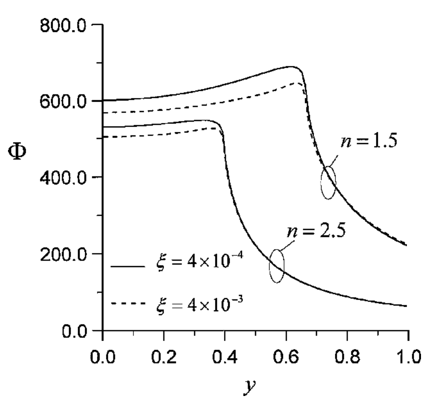

Fig. 7 Effects of refractive index n on the dimensionless radiative heat source distributions in the case of $N = 1$, $\kappa = 0.001$, $\tau_0 = 1.0$, $T_{sur} = 2000$ K, and $T_0 = 300$ K.

of particles or droplets during radiant heating because the interior of particle is the first to reach the phase change temperature and undergoes phase change or overheating. Also, this phenomenon is not found in a homogeneous semitransparent slab irradiated uniformly, in which the radiation absorption of the homogeneous nonscattering layer close to the slab surface is always greater than that of the slab center due to Beer's law.

Figure 6 shows the dimensionless radial temperature distributions corresponding to Fig. 5. For comparison, the dimensionless temperature distributions when the volume radiation was treated as a surface interaction are also shown in Fig. 6. As shown in Fig. 6, for the particle or droplet with small optical thickness, the inner temperature is larger than the surface temperature. Treating the volume radiation as a surface interaction results in a large error for temperature distribution, especially in the case of small optical thickness. For the particle or droplet with large optical thickness, the error is mainly located in a site close to the surface. Only in the case of $\tau_0 \gg 1$ can the volume radiation be treated as a surface interaction.

The effects of refractive index n on the dimensionless radiative heat source and temperature distributions in the case of $N = 1$, $\kappa = 0.001$, $\tau_0 = 1.0$, $T_{sur} = 2000$ K, and $T_0 = 300$ K are shown in Figs. 7 and 8. Because refraction focuses the incoming rays close to the particle center, with the increases of the refractive index n, in the initial period the peak of dimensionless radiative heat source and temperature moves forward to the particle's center.

Figure 9 shows the dimensionless transient temperature evolutions in the case of $n = 1.5$, $\kappa = 0.001$, $\tau_0 = 1.0$, $N = 10$, $T_{sur} = 2000$ K, and $T_0 = 300$ K. Treating the volume radiation as a surface interaction results in a large error for the dimensionless temperature evolution curves. As shown in Fig. 9, when the radiation is treated as volume processes, the inner temperature is larger than the surface temperature, but when the radiation is treated as surface processes, the inner temperature is less than the surface temperature.

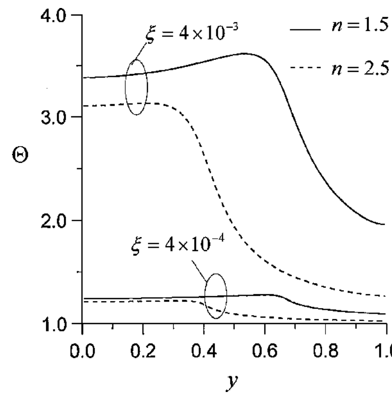

Fig. 8 Effects of refractive index n on the dimensionless temperature distributions in the case of $N = 1$, $\kappa = 0.001$, $\tau_0 = 1.0$, $T_{sur} = 2000$ K, and $T_0 = 300$ K.

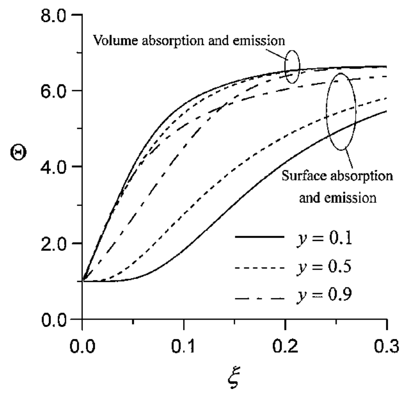

Fig. 9 Dimensionless transient temperature evolutions in the case of $n = 1.5$, $\kappa = 0.001$, $\tau_0 = 1.0$, $N = 10$, $T_{sur} = 2000$ K, and $T_0 = 300$ K.

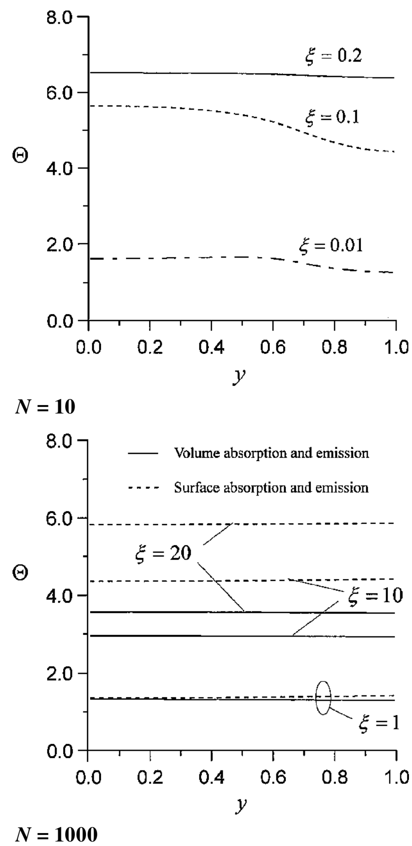

Fig. 10 Influence of conduction-to-radiation parameter N on the dimensionless radiative heat source and temperature distributions in the case of $n = 1.5$, $\kappa = 0.001$, $\tau_0 = 1.0$, $T_{sur} = 2000$ K, and $T_0 = 300$ K.

This mainly results from the different distribution of radiative soure term when different treatments of radiation processes are used.

The conduction-to-radiation parameter N characterizes the relative importance of conduction with regard to radiation. Figure 10 shows the influences of conduction-to-radiation parameter on the dimensionless radiative heat source and temperature distributions in the case of $n = 1.5$, $\kappa = 0.001$, $\tau_0 = 1.0$, $T_{sur} = 2000$ K, and $T_0 = 300$ K. As shown in Fig. 10, with an increase of conduction-to-radiation parameter, the dimensionless radiative heat source and temperature distributions become more uniform. Although, when $N = 1000$, for the particle with small optical thickness, due to the different distribution of the radiative source term, the computed results of volume radiation differ significantly from those of surface radiation.

Conclusions

A method to analyze the transient coupled radiation–conduction in a semitransparent spherical particle surrounded by isothermal black walls was developed. The radiative transfer coefficients were deduced using the ray tracing method in combination with Hottel and Sarofim's zonal method.[14] The radiative heat source term was calculated by the radiative transfer coefficients, and the transient energy equation was solved by an implicit finite difference method. The effects of the related parameters on the transient radiative heat source and temperature distribution were analyzed. The main conclusions can be summarized as follows:

1) The peak of dimensionless radial radiative heat source can be located in the interior shell of the particle or droplet with small optical thickness when heated by surrounding radiation.

2) Treating the volume radiation as a surface interaction results in large error of transient temperature distribution for the case of small optical thickness. The volume radiation can be treated as a surface interaction only in the case of $\tau_0 \gg 1$.

3) With increases in the conduction-to-radiation parameter, the dimensionless radiative heat source and temperature distributions become more uniform. Although, when $N = 1000$, for the particle with small optical thickness, due to the different distribution of the radiative source term, the computed results of volume radiation differ significantly from those of surface radiation.

Acknowledgments

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